#2 $u(x,t) = \underbrace{\frac{1}{2}\left[\phi(x+2t) + \phi(x-2t)\right]}_{①} + \underbrace{\frac{1}{2\cdot 2}\int_{x-2t}^{x+2t} \chi e^{-\chi^2}\, d\chi}_{②} + \underbrace{\frac{1}{2\cdot 2}\iint_{\Delta} f}_{③}$

Part ① $= e^{-(x+2t)^2} + e^{-(x-2t)^2}$

Part ② $= \frac{1}{-2}\, e^{-x^2}\Big|_{x-2t}^{x+2t}$

$$= \frac{1}{2}\left[e^{-(x-2t)^2} - e^{-(x+2t)^2}\right]$$

Part ③ $= \int_0^t \int_{x-(2t-s)}^{x+(2t-s)} x t e^{-x^2}\, dy\, ds$

$$= \int_0^t \frac{t}{2}\, e^{-x^2}\Big|_{x+(2t-s)}^{x-(2t-s)}\, ds$$

$$= \int_0^t \frac{t}{2}\left[e^{-(x-2t+s)^2} - e^{-(x+2t-s)^2}\right] ds$$

$P_1 = x-2t+s$ $\qquad$ $P_2 = x+2t-s$

$\frac{\partial P_1}{\partial s} = 1$ $\qquad$ $\frac{\partial P_2}{\partial s} = -1$

$P_1\big|_t = x-t$ $\qquad$ $P_2\big|_t = x+t$

$$= \frac{t}{2}\int_0^{x-t} e^{-P_1^2}\, dP_1 + \frac{t}{2}\int_0^{x+t} e^{-P_2^2}\, dP_2$$

$$= \frac{t}{2}\left[\frac{\sqrt{\pi}}{2}\,\mathrm{Erf}(x-t) + \frac{\sqrt{\pi}}{2}\,\mathrm{Erf}(x+t)\right]$$

$u(x,t) = \frac{1}{2}① + \frac{1}{4}② + \frac{1}{4}③$

$$= \frac{1}{2}e^{-(x+2t)^2} + \frac{1}{2}e^{-(x-2t)^2} + \frac{1}{8}e^{-(x-2t)^2} - \frac{1}{8}e^{-(x+2t)^2} + \frac{1}{4}③$$

$$= \frac{3}{8}e^{-(x+2t)^2} + \frac{5}{8}e^{-(x-2t)^2} + \frac{t\sqrt{\pi}}{16}\left[\mathrm{Erf}(x-t) + \mathrm{Erf}(x+t)\right]$$