⑨

P3 #1(b) For Neumann boundary condition, we take even continuation of h to $x<0$:

$$H(x) = \begin{cases} 1 & x>0 \quad \text{(given)} \\ 1 & x<0 \quad \text{(even)}. \end{cases}$$

Similarly.

① For $x > 2t$, $u(x,t) = \frac{1}{4}\int_{x-2t}^{x+2t} H(y)\,dy = t$

② For $x \in (0, 2t)$, $u(x,t) = \frac{1}{4}\int_{x-2t}^{x+2t} H(y)\,dy$

$$= \frac{1}{4}\int_{0}^{x+2t} 1\,dy + \frac{1}{4}\int_{x-2t}^{0} 1\,dy$$

$$= t$$

$\Rightarrow$ General solution:

$$u(x,t) = \begin{cases} t & x > 2t \\ t & 0 < x < 2t \end{cases}$$