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P3 D'Alembert Formula:

Consider wave equation in domain $\{x>0, t>0\}$, initial conditions, and a boundary condition

$$u_{tt} - c^2 u_{xx} = f(x,t) \qquad x>0,$$
$$u|_{t=0} = g(x) \qquad x>0$$
$$u_t|_{t=0} = h(x) \qquad x>0.$$
$$u|_{x=0} = 0$$
$$(\text{or } u_x|_{x=0} = 0)$$

Consider first IVP on the whole line:

$$u_{tt} - c^2 u_{xx} = F(x,t) \qquad x>0$$
$$u|_{t=0} = G(x) \qquad x>0$$
$$u_t|_{t=0} = H(x) \qquad x>0$$

D'Alembert formula

$$u(x,t) = \frac{1}{2}\left(G(x+ct) + G(x-ct)\right) + \frac{1}{2c}\int_{x-ct}^{x+ct} H(x')dx'$$

$$+ \frac{1}{2c}\int_0^t \int_{x-c(t-t')}^{x+c(t-t')} F(x',t')dx'dt'$$

and we need to take $0<x<ct$, resulting for

$f=0 \Rightarrow F=0 \qquad g=0 \Rightarrow G=0$

$\Rightarrow$ in this case,

$$u(x,t) = \frac{1}{2c}\int_{x-ct}^{x+ct} H(x')dx'$$