

$$E(t) = \frac{1}{2} \int_0^L (u_t^2 + Ku_{xx}^2 + \omega^2 u^2) dx$$

Take the derivative with respect to t and then integrate the middle term by parts.

$$\begin{aligned} E'(t) &= \frac{1}{2} \int_0^L (u_{tt}u_t + Ku_{xx}u_{xxt} + \omega^2 uu_t) dx \\ &\quad K \left(u_{xt}u_{xx} \Big|_0^L - \int_0^L u_{xt}u_{xxx} dx \right) \\ &\Rightarrow K \left(u_{xt}u_{xx} \Big|_0^L - u_t u_{xxx} \Big|_0^L + \int_0^L u_t u_{xxxx} dx \right) \\ E'(t) &= \int_0^L u_t (u_{tt} + Ku_{xxxx} + \omega^2 u) dx + K [u_{xt}u_{xx} - u_t u_{xxx}]_0^L \\ &= K [u_{xt}u_{xx} - u_t u_{xxx}]_0^L \end{aligned}$$

After plugging in the bounds and checking boundary conditions, we see that $E'(t) = 0$ and thus $E(t)$ is a constant function, independent of t .