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P3 #(a) For Dirichlet boundary condition, we take odd continuation of h to $x<0$:

$$H(x) = \begin{cases} 1 & x>0 \quad \text{(given)} \\ 0 & x=0 \\ -1 & x<0 \quad \text{(odd)} \end{cases}$$

In this case, it's easy to see that $c=2$

so $\begin{cases} x-2t<0 \Rightarrow 0<x<2t \\ x-2t>0 \Rightarrow x>2t \end{cases}$

① For $0<x<2t$,

$$u(x,t) = \frac{1}{4}\int_{x-2t}^{x+2t} H(y)\,dy$$

$$= \frac{1}{4}\int_{0}^{x+2t} 1\,dy + \frac{1}{4}\int_{x-2t}^{0} (-1)\,dy$$

$$= \frac{1}{4}(x+2t) + \frac{1}{4}\left((-y)\Big|_{x-2t}^{0}\right)$$

$$= \frac{1}{4}(x+2t) + \frac{1}{4}(0+x-2t)$$

$$= \frac{x}{2}$$

② For $x>2t$,

$$u(x,t) = \frac{1}{4}\int_{x-2t}^{x+2t} H(y)\,dy = \frac{1}{4}\int_{x-2t}^{x+2t} 1\,dy = t$$

$\Rightarrow$ general solution:

$$u(x,t) = \begin{cases} t & , x>2t \\ \frac{x}{2} & , 0<x<2t \end{cases}$$