(11)

P3 #1(d) For Neumann Boundary Condition, we take even continuation of h to $x<0$:

$$H(x) = \begin{cases} x & x>0 \\ -x & x<0 \end{cases}$$

Similarly,

① For $x>2t$, $u(x,t) = \frac{1}{4}\int_{x-2t}^{x+2t} H(y)\,dy$

$$= \frac{1}{4}\int_{x-2t}^{x+2t} y\,dy$$

$$= xt$$

② For $0<x<2t$, $u(x,t) = \frac{1}{4}\int_{x-2t}^{x+2t} H(y)\,dy$

$$= \frac{1}{4}\int_{0}^{x+2t} y\,dy + \frac{1}{4}\int_{x-2t}^{0} (-y)\,dy$$

$$= \frac{1}{8}(x+2t)^2 + \frac{1}{8}(x-2t)^2$$

$$= \frac{1}{4}x^2 + t^2$$

$\Rightarrow$ General Solution:

$$u(x,t) = \begin{cases} xt & x>2t \\ \frac{1}{4}x^2+t^2 & 0<x<2t \end{cases}$$