a) Find the general solution of

$$y'' + 4y = \frac{1}{\cos 2t}, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

b) Find solution, such that $y(0) = 0$, $y'(0) = 0$

$$r^2 + 4 = 0$$
$$r = \pm 2i$$
$$y = c_1 \cos 2t + c_2 \sin 2t$$

$$W = \begin{vmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{vmatrix} = 2$$

$$W_1 = \begin{vmatrix} 0 & \sin 2t \\ 1 & 2\cos 2t \end{vmatrix} = -\sin 2t$$

$$W_2 = \begin{vmatrix} \cos 2t & 0 \\ -2\sin 2t & 1 \end{vmatrix} = \cos 2t$$

$$y_p(t) = \cos 2t \int \frac{\frac{-\sin 2t}{\cos 2t}}{2}\, dt + \sin 2t \int \frac{\frac{\cos 2t}{\cos 2t}}{2}\, dt$$

$$= \cos 2t \int \frac{\frac{-\cancel{2}\sin t \cos t}{\cos^2 t}}{\cancel{2}}\, dt + \sin 2t \int \frac{\frac{2\cos^2 t - 1}{\cos^2 t}}{2}\, dt$$

$$= -\cos 2t \int \frac{\sin t}{\cos t}\, dt + \sin 2t \int \frac{2 - \frac{1}{\cos^2 t}}{2}\, dt$$

$$= -\cos 2t\,(-\ln|\cos t|) + \tfrac{1}{2}\sin 2t\,(2t - \tan t)$$

$$= \cos 2t \ln|\cos t| + t\sin 2t - \tfrac{1}{2}\sin 2t \tan t$$

$$y = c_1 \cos 2t + c_2 \sin 2t + \cos 2t \ln|\cos t| + t \sin 2t - \tfrac{1}{2}\sin 2t \tan t$$

$$y' = -2c_1 \sin 2t + 2c_2 \cos 2t - 2\sin 2t \ln|\cos t| - \cos 2t \frac{\sin t}{\cos t}$$
$$+ \sin 2t + 2t\cos 2t - \cos 2t \tan t - \tfrac{1}{2}\sin 2t \sec^2 t$$

$$y(0) = c_1 + \ln(1) = 0 \qquad c_1 = 0$$

$$y'(0) = 2c_2 = 0 \qquad c_2 = 0$$

$$y = \cos 2t \ln|\cos t| + t \sin 2t - \tfrac{1}{2}\sin 2t \tan t$$