(12)

P4 #(a)

Since $e = \frac{1}{2}(u_t^2 + u_x^2)$ & $p = u_t u_x$

$$\Rightarrow \frac{\partial e}{\partial t} = \frac{1}{2}\left(2u_t \cdot u_{tt} + 2u_x u_{xt}\right) = u_t u_{tt} + u_x u_{xt}$$

$$\frac{\partial e}{\partial x} = \frac{1}{2}\left(2u_t \cdot u_{tx} + 2u_x u_{xx}\right) = u_t u_{tx} + u_x u_{xx}$$

$$\frac{\partial p}{\partial t} = u_{tt} u_x + u_t u_{xt}$$

$$\frac{\partial p}{\partial x} = u_{tx} u_x + u_t u_{xx}$$

Since 1D Wave equation is

$$u_{tt} - c^2 u_{xx} = 0$$

And $c = 1$

$$\rightarrow u_{tt} = u_{xx}$$

Also, since $u_{xt} = u_{tx}$,

$$\Rightarrow \frac{\partial e}{\partial t} = u_t u_{tt} + u_x u_{xt} = u_t u_{xx} + u_x u_{tx} = \frac{\partial p}{\partial x}$$

$$\frac{\partial e}{\partial x} = u_t u_{tx} + u_x u_{xx} = u_t u_{xt} + u_x u_{tt} = \frac{\partial p}{\partial t}$$